Exploring Mandelbrot Fractals with MMSFORTH

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Abstract

Mandelbrot fractals became generally known in 1985 and instantly fasci-
nated mathematicians, artists and all those who marvel at the richness and
beauty which results from the plotting of these relatively simple equations.
Initially, the generation of a sufficiently detailed plot would require calcula-
tion on a large computer, or an overnight run. However, combining standard
MMSFORTH for IBM PC with three of its optional extension wordsets --
floating-point QUANs, long addressing, and 8087 fast-math support -- we
are able to generate 200x200-pixel Mandelbrot plots of typically 80 itera-
tions at about 10 minutes per plot.

Background:

The August 1985 issue of Scientific American, with its colorful, intricate
cover picture of a plot of a simple mathematical function, inspired MMS to
duplicate these results on personal computers. Independently, several users
of MMSFORTH (and at least one BASIC programmer we know) produced
programs which could create Mandelbrot plots displayed to screen. One TRS-
80 owner, lacking sufficient screen resolution, found a good way to display
to a color printer instead. Initial methods took 8 to 12 hours to produce a
single plot. This inspired several interesting methods of skimping on the
number of points calculated, in order to see if an area would yield interest-
ing results before spending the time on a full plot.

Explanation of the Mandelbrot Set plot:

The Mandelbrot Set is the plot of a set of points in a complex plane. By
complex, we mean the mathematical definition of a set of numbers with a
real and an imaginary component. In the plot, the x (horizontal) axis is the
real part and the y (vertical) axis is the imaginary part. At each point an
iterative formula is applied to the complex number defined by the x and y
axes, until either an assumed maximum number of iterations is reached, or
the value of the complex number goes to infinity. (Infinity is here defined
as any case where the complex value exceeds 2, because it is a character-
istic of the formula used that if the value goes over 2 it will go to
infinity.) A color is assigned to the point, based on the number of iterations
required before the point goes to infinity. Numbers which do not go to
infinity within the chosen number of iterations are assigned, by convention,
to black. The formula used to discover the Mandelbrot Set is $z_n = (z_{n-1})^2 +$
c, where the initial value z_0 is always zero and c is the complex number.

The actual Mandelbrot Set is comprised of those points which remain finite, but the term is loosely used to describe the entire plot. This deceptively simple equation produces the wonderfully intricate plots shown in the Scientific American article and here.

The initial Mandelbrot fractal plot (Figure 1) typically is displayed from real coordinates -2.0 on the left to +0.5 on the right, and imaginary coordinates -1.25 on the bottom to +1.25 at the top. Like other types of fractal images, the details of its image are generated from its fundamental shape. Bulges on the edge of the larger circle can be seen to closely resemble the "snowman" shape of the main figure itself.

Considered from an artistic standpoint, the most interesting figures are at the edges of the "black" area, and from a mathematical standpoint the good news is that one can zoom in on these areas of interest indefinitely. Although each single image is limited by the resolution of the screen or printer, the ultimate magnification is only limited by the precision of the computer's arithmetic.

The remaining figures were generated with our Mandelbrot program, by setting closer coordinate ranges and by selecting an interesting maximum number of iterations for the point-by-point calculations. Figure 2, for example, is a close-up view at 120 iterations of that part of the basic Mandelbrot plot which extends only from -1.075 on the left to -1.043 on the right, and from +0.25 on the bottom through +0.28045 at the top.

In short, here is a rich and diverse universe to be explored! Although this universe is not visible to the eye without computer processing, it is truly natural in the sense that it derives from a very simple mathematical equation which, although only recently discovered, was here all along.

Hitching Up Existing MMSFORTH Routines:

At MMS we replaced the earlier scaled-integer or ROM-floating-point calculations with the MMSFORTH 8087 floating-point command set. We already had complex functions coded in our MMS 8087 wordset, so it only required changing two routines for complex-squaring and complex value to call their appropriate MMS-8087 words, plus use of the 8087 version of MMS-FORTH's number input routine. These modifications were minor and worked beautifully! Additional, conventional routines were used to store resulting RAM images to disk, to load disk images back into memory, and to dump a screen image to dot-addressable printers.

Our initial plots, like the ones displayed in the Scientific American article, always assigned a range of values to each color. However, with only four colors on the IBM PC (including black!) we have since improved the plotting technique by simply picking a number of iterations of resolution and cycling through the same colors over and over (i.e., cycle to the next color every five iterations, etc.). This gives a richer set of contours than would be available otherwise, and even works in black and white.

Results

    Generation time has been reduced to 10-15 minutes per 200x200-pixel
plot with 100 iterations, and 20-30 minutes per plot with 200 iterations.
Plots of this sort provide sufficient resolution to be of great interest. Plots
which are unusually rich in "black space" will take longer because each
black dot must run through the maximum number of iterations.

Conclusions:

    Mandelbrot fractal plots are of great interest. With the help of an 8087
chip and a good set of complex arithmetic routines, they may be produced
quickly on the popular IBM PC and similar personal computers.

Credits & References:

"Exploring the Mandelbrot Set" -- Computer Recreations, by A. K. Dewdney,
Scientific American, August 1985, pp. 16-20.

"Simplified Mandelbrot Calculations", by James Gerow, Newsletter of the
MMSFORTH User Group of Eastern Massachusetts (for MMSFORTH Users
only), October 1985.

The Fractal Geometry of Nature, by Benoit B. Mandelbrot (W. H. Freeman &
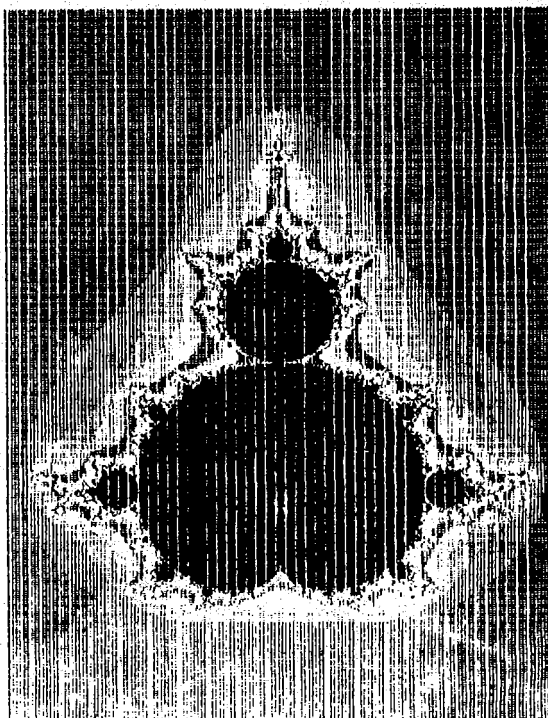Co., New York, 1983).

Figure 1 - **Initial Mandelbrot Fractal Plot** ("The Snowman")
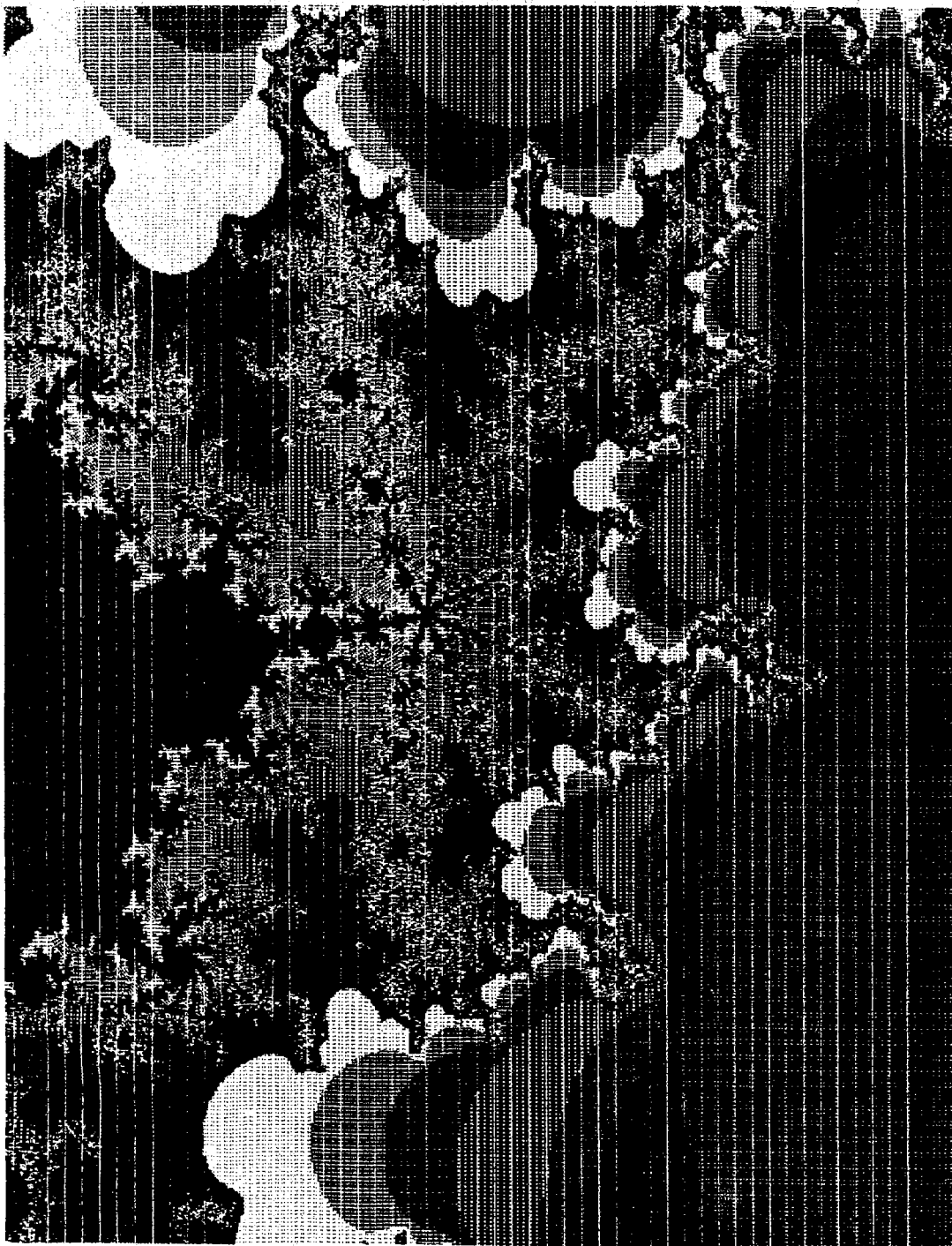(from -2.0,-1.25 to +0.5,+1.25; 128 iterations)

Figure 2 - Detail of Mandelbrot Plot (A Zoomed View)